RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, SEPTEMBER 2020
FIRST YEAR [BATCH 2019-22]

Date : 25/09/2020 **MATHEMATICS(Honours)**
Time : 11am – 3pm **Paper** : MACT-3 Full Marks : 25

Instructions to the students

- Write your College Roll No, Year, Subject and Paper Number on the top of the Answer Script.
- Write your Name, College Roll No, Year, Subject and Paper Number on the text box of your e-mail.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
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- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers on single side and they must give a minimum 1 inch margin at the left side of each paper.
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Group - A (Classical Algebra)

Answer any 2 questions from question numbers 1-3. [2 x 3 = 6 marks]

1. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \alpha^2\beta^2$.
2. Use the following hint to solve the equation :

$$x^3 + 6x^2 + 11x + 6 = 0.$$

(Hint : The roots are in arithmatic progression)

3. Solve : $x^4 - 8x^3 + 17x^2 - 8x + 1 = 0$.

Answer any 1 question from question numbers 4-5. [6 marks]

4. Let $a_i, i = 1, 2, .., n$ be positive real numbers and $s = a_1 + a_2 + ... + a_n$, then show that

(a) $\displaystyle\prod_{i=1}^{n}(s - a_i) < \frac{s}{2}$. [3]

(b) $2s < \prod_{i=1}^{n}(s + a_i)$. [3]

5. (a) Let z be a complex number satisfying the equation $\left|z - \frac{12}{z}\right| = 1$. Find the greatest and least values of $|z|$. [3]

   (b) Using De Moivre's theorem express $\cos[(n+1)\theta]$, $n \in \mathbb{N}$, in terms of $\cos\theta$ and $\sin\theta$. (Hint : use binomial expansion) [3]

Group - B (Analysis-II)

Answer any 2 questions from 6-8 in this group. [2 x 6.5 = 13 marks]

6. (a) Prove: $\sin x \geq \frac{2x}{\pi}$, $\forall\, x \in \left[0, \frac{\pi}{2}\right]$. [3.5]

   (b) Let f be continuous on $[0, 2]$ and twice differentiable on $(0, 2)$. Show that if $f(0) = 0, f(1) = 1$ and $f(2) = 2$, then $\exists\, x_0 \in (0, 2)$ such that $f''(x_0) = 0$. [3]

7. (a) Prove or disprove the following statement with proper justification: [4]
   There does not exist any continuous surjection $f : \mathbb{R} \to \mathbb{R}$, which attains each of its functional values exactly two times.

   (b) A function $f : \mathbb{R} \to \mathbb{R}$ is continuous at zero and satisfies the following conditions:

$$f(0) = 0 \text{ and } f(x + y) \leq f(x) + f(y),\ \forall\, x, y \in \mathbb{R}.$$

   Prove that f is uniformly continuous on $\mathbb{R}$. [2.5]

8. (a) Determine all real values of x for which the following series converges: [4]

$$\sum_{n=1}^{\infty}\left(1 + \frac{1}{2} + ... + \frac{1}{n}\right)\frac{\sin(nx)}{n}.$$

   (b) Show that the convergence of $\sum_{n=1}^{\infty} a_n$ implies the convergence of [2.5]

$$\sum_{n=1}^{\infty}\frac{\sqrt{a_n}}{n},\ \text{if } a_n \geq 0, \forall n.$$

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Group - C (Linear Algebra I)

Answer any 3 questions from 9-13 in this group. [3 x 5 = 15 marks]

9. Let $\mathcal{S}$ be the set of all linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^2$. Show that $\mathcal{S}$ is a vector space over the field $\mathbb{R}$ with respect to the following operations :

$$\text{Addition } : (T_1 + T_2)(\alpha) = T_1(\alpha) + T_2(\alpha), \text{ and scalar multiplication } : (cT_1)(\alpha) = c(T_1(\alpha)),$$

where $T_i \in S$, $c \in \mathbb{R}$ and $\alpha \in \mathbb{R}^3$. What will be the dimension of $\mathcal{S}$? If $\mathcal{S}$ is a finite dimensional vector space, then give a basis of $\mathcal{S}$. [3+1+1]

10. Let V be a finite dimensional vector space and $T : V \to V$ be a linear transformation.

    (a) Show that $ker(T^k) \subseteq ker(T^{k+1}), \forall k \in \mathbb{N}$. [1]

    (b) If $z \in ker(T^{k+1}) \setminus ker(T^k)$, for some fixed k, then show that $\{z, Tz, T^2z, ..., T^kz\}$ is a linearly independent collection of vectors in V. [4]

11. Let $M_n(\mathbb{R})$ denote the vector space of all $n \times n$ matrices with real entries and $S \subset M_n(\mathbb{R})$ be the set of all symmetric $n \times n$ matrices

(a) Show that S form a vector subspace of $M_n(\mathbb{R})$. [1]

(b) Find the complement subspace of S. [2]

(c) Is there any direct sum representation of $M_n(\mathbb{R})$, without involving S as one of the direct sum pair? Justify your answer. (Note : no marks will be awarded without justification) [2]

12. (a) Using elementary row operations find the inverse of [1.5 + 1]

$$A = \begin{pmatrix} 6 & -2 & 12 \\ 2 & 1 & 6 \\ 7 & -2 & 15 \end{pmatrix}$$

Use this inverse to solve the linear system $AX = b$, where $b = (1, 2, 3)^t$ is a column vector.

(b) Express the inverse found in part (a) as the product of elementary matrices. [2.5]

13. Let A and B be two $n \times n$ matrices. Prove or disprove the following statements

(a) $A^2 = A$ implies either $A = I$ or $A = 0$. [1.5]

(b) If $AB = 0$, then there is a non-trivial solution of the linear system $BAv = 0$. [1]

(c) If there exists a linearly independent set of vectors $S = \{v_1, v_2, ...v_k\}$, where $k < n$, such that the set $W = \{Av_i : 1 \leq i \leq k\}$ is linearly dependent, then $det(AB) = 0$ [2.5]

Group - D (Applications of Differential Calculus)

Answer any 2 questions from 14-16 in this group. [2 x 5 = 10 marks]

14. Prove that the locus of the extrimity of the polar subnormal of the curve $r = f(\theta)$ is $r = f'(\theta - \pi/2)$. Show that this locus is an equiangular spiral if the curve is an equiangular spiral $r = ae^{\theta cot\alpha}$. [5]

15. If ρ, ρ' be the radii of curvature at the ends of two conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that [5]

$$\left(\rho^{2/3} + \rho'^{\,2/3}\right)(ab)^{2/3} = a^2 + b^2.$$

16. Show that the four asymptotes of the curve :

$$(x^2 - y^2)(y^2 - 4x^2) + 6x^3 - 5x^2y - 3xy^2 + 2y^3 - x^2 + 3xy - 1 = 0$$

cut the curve in eight points which lie on the circle : $x^2 + y^2 = 1$. [5]

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